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called a rational space from a continuous space."

In my *Rational Geometry* I treat Non-Euclidean Spherics without any continuity assumption whatsoever.

PROBLEMS FOR SOLUTION.

ALGEBRA.

E. Kesner, Salida, Col., solved 208 and 209.

211. Proposed by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Prove that $p-qx$ and $q-px$ tend to equality as x dimishes to zero, but yet that their limits are not equal. [Edwards' *Differential Calculus*, p. 7, ex. 10.]

Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

The limits of $p-qx$ and $q-px$ as x dimishes to zero, are p and q. Suppose $p>q$, then by subtraction the difference at the limit is $p-q$ and the difference for any value of x is $p-q-qx+px$ which evidently decreases as x diminishes. Likewise for $q>p$. Therefore $p-qx$ and $q-px$ tend to equality as x diminishes to zero.

Also solved by G. B. M. Zerr, and J. Scheffer.

212. Proposed by F. P. MATZ, Ph. D., Sc. D.

Solve $\dfrac{x^2+2x+3}{x^2-2x+3}+\dfrac{x^2-2x+3}{x^2+2x+3}=\dfrac{10}{3}$.

I. Solution by O. S. WESTCOTT, Waller High School, Chicago, Ill., ELMER SCHUYLER, Brooklyn, N. Y., and J. SCHEFFER, Hagerstown, Md.

If we write $\dfrac{x^2+2x+3}{x^2-2x+3}=y$, then will $y+\dfrac{1}{y}=\frac{10}{3}$, and $y=3$ or $\frac{1}{3}$.

a) If $\dfrac{x^2+2x+3}{x^2-2x+3}=3$, $x=3$ or 1.

b) If $\dfrac{x^2+2x+3}{x^2-2x+3}=\frac{1}{3}$, $x=-3$ or -1.

II. Solution by A. H. HOLMES, Brunswick, Maine.

Clearing the given equation of fractions and reducing, we obtain $x^4-10x^2=-9$, whence $x=1$, -1, 3, -3.

Also solved by G. W. Greenwood. E. L. Rich, M. E. Graber, G. B. M. Zerr, F. D. Posey, S. S. Flory, L. E. Newcomb, and E. Kesner.

213. Proposed by F. P. MATZ, Ph. D., Sc. D.

Find the two roots of the equation $x^5-209x+56=0$, whose product is unity.

I. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill., and F. D. POSEY, A. B., San Mateo, Cal.

The equation may be written $(x^2-4x+1)(x^3+4x^2+15x+56)=0$. Hence two of the roots are $2\pm\sqrt{3}$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let y and $1/y$ be the roots, then

$$y^5-209y+56=0 \dots\dots\dots (1),$$
$$56y^5-209y^4+1=0 \dots\dots\dots (2).$$

56 times (1)−(2) gives $209y^4-56\times209y+56^2-1=0$, or $209(y^4-56y+15)=0$.

$\therefore y^4-56y+15=0 \dots\dots\dots (3)$.

(1)−56 times (2) gives, after dividing by y, $15y^4-56y^3+1=0 \dots\dots\dots (4)$.

15 times (3)−(4) gives $y^3-15y+4=0 \dots\dots\dots (5)$.

(3)−15 times (4) gives $4y^3-15y^2+1=0 \dots\dots\dots (6)$.

4 times (5)−(6) gives $y^2-4y+1=0$. $\therefore y=2\pm\sqrt{3}$.

Hence the roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Also solved by A. H. Holmes, J. Scheffer, Lloyd Holsinger, L. E. Newcomb, and Elmer Schuyler.

214. Proposed by Editor EPSTEEN.

How many terms are there in the expansion of $(x_1+x_2+\dots\dots\dots+x_n)^n$?

I. Solution by F. OWENS, M.A.; Mrs. F. OWENS, M.A.; G. B. M. ZERR, A. M., Ph. D.; G. W. GREENWOOD, M. A.

Let ${}_nu_r$ denote the number of terms in the rth power of an expression consisting of n terms. By considering it in the form

$$[x_1+(x_2+x_3+\dots\dots\dots+x_n)]^r$$

we see that

$${}_nu_r=1+{}_{n-1}u_1+{}_{n-1}u_2+\dots\dots\dots+{}_{n-1}u_r.$$

$\therefore {}_nu_r={}_nu_{r-1}+{}_{n-1}u_r$.

Hence we may write down the number of terms in any power of any expression thus:

1	1	1	1	1	....
1	2	3	4	5	....
1	3	6	10	15	....
1	4	10	20	35	....
.....	.....	.....	.....	.....	....

where each element is obtained by adding the one the left to the one above it.

It can be seen, from Pascal's Triangle, that the number of terms is given by the expression for the $(r+1)$st terms of the nth order of figurate numbers, ${}_nu_r=\frac{(n+r-1)!}{r!\,(n-1)!}$. In particular ${}_nu_n=\frac{(2n-1)!}{n!(n-1)!}$.

II. Solution by J. SCHEFFER, Hagerstown, Md.

The problem reduces to finding the number of products of n quantities and their power. By expanding, $\frac{1}{1-x_1x}$, $\frac{1}{1-x_2x}$, $\frac{1}{1-x_3x}$, etc., into series, we find,

$$\frac{1}{1-x_ix}=1+x_ix+x_i^2x^2+x_i^3x^3+\dots\dots \quad (i=1,\ 2,\ \dots\dots,\ n).$$

$\left(\frac{1}{1-x_1x}\right)\left(\frac{1}{1-x_2x}\right)\left(\frac{1}{1-x_3x}\right)\dots\dots=1+s_1x+s_2x^2+s_3x^3+\dots\dots$ where $s_1=x_1+x_2+\dots\dots x_n$, $s_2=x_1^2+x_2^2+x_3^2\dots\dots+x_1x_2+x_1x_3\dots\dots+x_2x_3\dots\dots$, $s_3=x_1^3+x_2^3+\dots\dots x_n^2$ $x_1^2x_2+x_1^2x_3+\dots\dots+x_1x_2x_3+\dots\dots$

To find the number of products in any of these sets of products, we put $x_1=x_2=x_3=\dots\dots=x_n=1$, and we get

$$\frac{1}{1-x_1x}\cdot\frac{1}{1-x_2x}\cdot\frac{1}{1-x_0x}\dots\dots=\frac{1}{(1-x)^n}=(1-x)^{-n}.$$

$\therefore s_n$ is equal to the coefficients of x^n in the expression of $(1-x)^{-n}$.

$\therefore s_n=\frac{n(n+1)(n+2)\dots\dots[n+(n-1)]}{n!}$, or, multiplying numerator and denominator by $(n-1)!$ we get $\frac{(2n-1)!}{(n-1)!n!}$ as the required result.

Also solved by L. E. Newcomb, and Elmer Schuyler.

GEOMETRY.

Two solutions of 238 were received from L. E. Newcomb, Los Gatos, Cal.

239. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Divide the sides of a triangle ABC internally in P, Q, R, so that $BP/PC=CQ/QA=AR/RB$. QR cuts BC externally in S. Show that BS is to CS in the duplicate ratio of CP to PB.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Draw SD parallel to AB; let m=any proper fraction, and let $PC=ma$, $QA=mb$, $RB=mc$. Then

$$\frac{BP}{PC}=\frac{a(1-m)}{am}=\frac{CQ}{QA}=\frac{b(1-m)}{bm}=\frac{AR}{RB}=\frac{c(1-m)}{cm}=\frac{1-m}{m}.$$

Considering AB, AC as the axes of coördinates the equation to RQ is

$$\frac{x}{bm}+\frac{y}{c(1-m)}=1,\ \text{equation to } BC \text{ is } \frac{x}{b}+\frac{y}{c}=1.$$